D-branes in the diagonal $SU(2)$ coset

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ABSTRACT: The symmetry preserving D-branes in coset theories have previously been described as being centered around projections of products of conjugacy classes in the underlying Lie groups. Here, we investigate the coset where a diagonal action of $SU(2)$ is divided out from $SU(2) \times SU(2)$. The corresponding target space is described as a (3-dimensional) pillow with four distinguished corners. It is shown that the (fractional) brane which corresponds to the fixed point that arises in the CFT description, is spacefilling. Moreover, the spacefilling brane is the only one that reaches all of the corners. The other branes are 3, 1 and 0 - dimensional.

Contents

1. Introduction

In a lagrangian description of CFT, one must impose boundary conditions for open world-sheets. These boundary conditions should be such that there is an action of the chiral symmetry algebra on open world-sheet correlators, in particular the resulting boundary CFT should be conformal. In free bosonic theories, this is achieved by simply imposing Neumann- or Dirichlet boundary conditions, cf. [1]. In curved backgrounds, the situation is more complicated. For instance in WZW models (describing strings propagating in Lie groups), the symmetry preserving D-branes are localized at certain conjugacy classes, cf. [2, 3].

On the other hand, one can describe D-branes from a perspective which is more oriented towards the algebraic description of CFT on the world-sheet. In that approach, one *begins* with imposing conformal invariance and other requirements on so-called boundary states [4]. Then, one can analyze the shapes of these boundary states. In free bosonic theories there is full agreement between these descriptions [5], but already in WZW models, the agreement is less direct [6].

A class of CFT models related to WZW models are the coset models. The relevant target spaces are adjoint quotients $G/\mathrm{Ad}(H)$, where $H \subset G$ is an embedded Lie subgroup of the connected, simply connected Lie group G, cf. [7, 8]. The D-branes in the lagrangian description are certain projected products of conjugacy classes [9]. In general, the CFT description of coset models involve a certain identification group [10 – 12]. In most of the literature on D-branes in such models, only situations where the identification group acts freely are considered, i. e. $\mathcal{G}_{\mathrm{id}}$ has no fixed points. The aim of the present note is to perform a geometric analysis of a model where such fixed points do occur, with interest primarily on the branes that correspond to the fixed point. We consider the sigma model with target space the coset

$$Q = \frac{\mathrm{SU}(2) \times \mathrm{SU}(2)}{\mathrm{Ad}(\mathrm{SU}(2))} , \tag{1.1}$$

which is the set of equivalence classes $[g_1, g_2]$ with the equivalence relation

$$(g_1, g_2) \sim (g g_1 g^{-1}, g g_2 g^{-1}),\qquad(1.2)$$

and with $g, g_1, g_2 \in \mathrm{SU}(2)$. The sigma model on Q can be described as a gauged WZW model [9], which after choosing levels $k, l, k + l \in \mathbb{Z}_{>0}$ for the three occuring factors, corresponds to a coset CFT. The particular choice $l = 1$ gives the series of Virasoro minimal models. In the sequel, we shall think of both l and k as being large, so that we can make use of the existing results on boundary conditions in coset models [9, 14] at large level.

In the next section, the geometry of the target space is described. The space Q is a pillow, or a pinched cylinder, with four distinguished corners. We introduce coordinates and classify degenerate points, for which the stabilizer of the coset action (1.2) is larger than at generic points. In the third and final section, the geometry of the subsets where the branes are centered is discussed. The dimensionalities of the branes are derived, and the branes reaching the corners of Q are classified.

2. Target space geometry

An adjoint diagonal coset (1.1) can be parametrized as follows. The set Q consists of equivalence classes

$$[g_1, g_2] = [g g_1 g^{-1}, g g_2 g^{-1}]\qquad g, g_1, g_2 \in G\,.\qquad(2.1)$$

For any $[g_1, g_2]$ we can choose a representative of $g_1 \sim t_1$ in $T_W = T/W$, where T is a maximal torus and W is the Weyl group. It remains to determine where we can choose a representative of g_2 with the remaining 'gauge freedom' S_{t_1}. Henceforth, G will denote $\mathrm{SU}(2)$. The stabilizers at the center $Z = \{\pm e\}$ are given by $S_{\pm e} = G$, and for other $t_1 \in T_W / Z$ we have $S_{t_1} = T$. This gives the fibration

$$Q = \left\{ (t, q) \ \mid\ t \in T_W\,, \quad \begin{array}{ll} q \in T_W & \text{if } t = \pm e \\ q \in G/\mathrm{Ad}(T) & \text{else} \end{array} \right\}.\qquad(2.2)$$

Using the Pauli matrices σ_i with relations $\sigma_i \sigma_j = \delta_{ij}\mathbf{1} + \mathrm{i}\epsilon_{ijk}\sigma_k$, one can parametrize group elements as

$$g(\psi, \theta, \phi) = \cos\psi\,\mathbf{1} + \mathrm{i}\sin\psi\,\bar{n}\cdot\bar{\sigma}\,.\qquad(2.3)$$

Here $\bar{n} = (\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta)$ is given by the standard spherical coordinates θ and ϕ, and T_W is the interval parametrized by $0 \le \psi \le \pi$. Note that we have

$$g^{-1} = g(-\psi, \theta, \phi) = g(\psi, \pi-\theta, \pi+\phi) = g^\dagger\,.\qquad(2.4)$$

The geometry of the parafermion coset $PF = G/\mathrm{Ad}(T)$ was described in [15] as a round disc. It can also be described as disc with two 'corners' [13]. Thus, we may think of Q as a pillow, or as a pinched cylinder [14] where neighboring corners are connected by four

singular lines. The description above has an arbitrariness in that we could have equally well fixed $\psi_2 \in [0, \pi]$. Then, we could have a pinched cylinder with the singular line going between other pairs of corners, instead. Either way, (1.1) has four corners. In the bulk, it is three-dimensional. We choose as coordinates the angles ψ_1, ψ_2 and θ, where θ may be either θ_1 or θ_2. Henceforth, let us take $\theta = \theta_2$ as in (2.2). At the edges of the pillow, $\psi_1, \psi_2 \in \{0, \pi\}$, the coordinate θ has no meaning. The ambiguity in θ is the only ambiguity, we cannot get rid of ψ_1 and ψ_2 as coordinates (assuming a fixed choice of coordinates on the covering). In any case, the coordinates ϕ_1 and ϕ_2 are gauged away.

Stabilizers

We can in a natural way think of Q as embedded in its covering $G \times G$, via the above described coordinates. The stabilizer of the Ad(G)-action on an element $(t, q) \in Q \subset G \times G$ is given by those $g \in G$ for which

$$(t, q) = (gtg^{-1}, gqg^{-1}), \iff g \in S_{(t,q)} = S_t \cap S_q. \tag{2.5}$$

For $t = \pm e$, $S_t = G$, else $S_t = T$. Since every element $q \in G$ can be brought to T via conjugation; $cqc^{-1} = t_q \in T$ with an element $c \in G$, one can show that the generic stabilizers are given by $S_q = c^{-1}Tc$, with $c \in G$ a fixed element (depending on q). Indeed, one may choose T to be any maximal torus, and one could equally well have defined $T = S_q$. Note that for $c \neq \pm e$, we have $c^{-1}Tc \cap T = Z$, the center of the group. The analysis is summed up in the following table.

$t \in$	$q \in$	S_t	S_q	$S_t \cap S_q$	$(t, q) \in Q$
Z	Z	G	G	G	corner
	$T \backslash Z$	G	T	T	edge/boundary
$T \backslash Z$	Z	T	G	T	boundary/edge
	$T \backslash Z$	T	T	T	boundary
	$G \backslash T$	T	cTc^{-1}	Z	interior

Note that the choice of the edges was arbitrary, so they must indeed have the same stabilizer as the rest of the boundary of Q (except for the corners). The corners are the most singular points, and have the largest stabilizer.

3. The projected conjugacy classes

In the large level limit, the D-branes in the coset Q are localized at sets $\pi_{\text{Ad}}(C_{abc})$ defined as follows, cf. [9, 13] for further details. First, introduce the following notation for the conjugacy classes,

$$C_x := \{g(\psi, \theta, \phi) \in G | \ \psi = x\}. \tag{3.1}$$

Second, define the operation $*$ on pairs of conjugacy classes,

$$C_x * C_y := \{g = g_x g_y \in G | \ g_x \in C_x, \ g_y \in C_y\}. \tag{3.2}$$

The set $C_x * C_y$ is invariant under $\mathrm{Ad}(G)$. Furthermore, introduce the sets

$$C_{abc} := (C_a \times C_b) * C_c \tag{3.3}$$
$$:= \{(g_a g_c, g_b g_c) \in G \times G|\ g_a \in C_a\,,\ g_b \in C_b\,,\ g_c \in C_c\} \subset G \times G\,.$$

These sets are the ones that project to the coset branes. Note that they are fixed under the adjoint action;

$$(g, g)\, C_{abc}\, (g^{-1}, g^{-1}) = C_{abc}\,. \tag{3.4}$$

By studying the sets C_{abc}, one can draw conclusions about the dimensionalities and other properties of the branes in Q. This is the topic of the rest of this note. First an observation. The simplest example of a set which is invariant under the diagonal Ad-action is the orbit of a point $(p_1, p_2) \in G \times G$. If this is a generic point (projecting to the bulk of Q), the stabilizer is $\mathbb{Z}_2$, thus the orbit $\mathrm{Ad}_G(p_1, p_2)$ is three-dimensional in $G \times G$. The geometric picture of this orbit is that (ψ_1, ψ_2) are fixed, (ϕ_1, ϕ_2) take all values and we have a line in the (θ_1, θ_2)-plane. If instead $(p_1, p_2) \in Z \times Z$, the orbit $\mathrm{Ad}_G(p_1, p_2)$ is zero- dimensional, and projects to a corner in Q. Thus, we shall need not only the dimensionalities of C_{abc} to find the dimensionalities of $\pi_{\mathrm{Ad}}(C_{abc})$, we also need the information about the stabilizers.

Corners

From (2.4) we learn that conjugacy classes $C_x \subset \mathrm{SU}(2)$ equal their own inverses; if $g \in C_x$, then $g^{-1} \in C_x$. For the $\psi_1 = 0$ edge to be contained in C_{abc} it is necessary that $a = c$ because otherwise, C_c does not contain the inverse of C_a. The equation (2.4) tells us that this condition also is sufficient. Thus, the edge defined by $\psi_1 = 0$ is reached if and only if $a = c$. For reaching the edge $\psi_1 = \pi$, we are faced with the following task: Given $g \in C_a$, we would like to have a $g' \in C_c$ such that $g(g')^{-1} = -1$. This is equivalent to that C_c contains the inverses of $(-1)C_a = C_{\pi-a}$. But then, $C_c = C_{\pi-a}$. As argued above, the edges per se are not interesting, but it is useful to note that a corner is where two edges meet (now viewing Q as a pillow, rather than a pinched cylinder). The analysis described above leads to the following conclusions about the four corners (whose θ-coordinates are irrelevant).

- $(0, 0) \in \pi_{\mathrm{Ad}}(C_{abc})$ if and only if $a = b = c$.

- $(0, \pi) \in \pi_{\mathrm{Ad}}(C_{abc})$ if and only if $(a = c$ and $b = \pi - c)$.

- $(\pi, 0) \in \pi_{\mathrm{Ad}}(C_{abc})$ if and only if $(a = \pi - c$ and $b = c)$.

- $(\pi, \pi) \in \pi_{\mathrm{Ad}}(C_{abc})$ if and only if $(a = \pi - c$ and $b = \pi - c)$.

In particular, if a brane contains two corners, it automatically contains all four. The brane which corresponds to the fixed point of the identification group is $\pi_{\mathrm{Ad}}(C_{fff})$, with $f = \pi/2$. This is the unique brane that contains all four corners, and is therefore in some sense the most degenerate brane. We shall show that the fixed point brane fills up all of Q. However, it is not the only brane that is three- dimensional.

Dimensionality of the fractional branes

It is instructive to first show that

$$C_f * C_f = G \,. \tag{3.5}$$

Note that (3.5) is equivalent to

$$\forall \; g \in G \,, \quad \exists \; u, w \in C_f \; : \; uw = g \,. \tag{3.6}$$

The product of two elements u and w on the equatorial conjugacy class

$$C_f := \left\{ g \in G \mid \psi_g = \frac{\pi}{2} \right\} \subset G \tag{3.7}$$

is computed using (2.3).

$$uw = \bar{n}_u \cdot \bar{n}_w \, \mathbf{1} - \mathrm{i} \, \bar{\sigma} \cdot (\bar{n}_u \times \bar{n}_w) \tag{3.8}$$

Take an arbitrary element $g \in G$. The following shows that there exist $u, w \in C_f$ such that $uw = g$. Choose $\bar{n}_u \in S^2$ and $\bar{n}_w \in S^2$ in the plane perpendicular to $\bar{n}_g$. Choose the angle between them such that $\bar{n}_u \cdot \bar{n}_w = \cos \psi_g$, and so that $\bar{n}_u \times \bar{n}_w$ points in the same direction as $\bar{n}_g$. Since the product uw is a group element, we do not even need to check that $|\bar{n}_u \times \bar{n}_w| = \sin \psi_g$, and hence $uw = g$. This proves the claim (3.5).

The following argument shows that

$$C_{fff} = G \times G \,, \tag{3.9}$$

which implies that the corresponding brane in the coset is spacefilling,

$$\pi_{\mathrm{Ad}} \left(C_{fff} \right) = Q \,. \tag{3.10}$$

First note that this is not an immediate consequence of (3.5), because the same factor $w \in C_f$ occurs at two places; we need to prove that for all $g_1, g_2 \in G$, we can choose $u, v, w \in C_f$ so that $uw = g_1$ and $vw = g_2$. A quick glance at (3.8) reveals that the choice of g_1 forces $\bar{n}_w \perp \bar{n}_1$. Given w, one may choose $u \in C_f$ such that $uw = g_1$. Now it remains to prove that with $\bar{n}_w$ fixed in the plane $\perp \bar{n}_1$ through the origin, and for any $g_2 \in G$, we can find $v \in C_f$ so that $vw = g_2$. We need $\bar{n}_w$ also to lie in the plane through the origin $\perp \bar{n}_2$. The two planes mentioned intersect at no less than two points on the sphere C_f. Choose $\bar{n}_w$ one of these points. Then choose $\bar{n}_v \perp \bar{n}_2$ such that $\bar{n}_w \cdot \bar{n}_v = \cos \psi_2$. It now follows $vw = g_2$. This proves the claim (3.9).

Dimensionality of generic branes

We proceed to determine the dimensions of $\pi_{ad}(C_{abf})$ for more general pairs a, b. The following argument shows that in the generic (nondegenerate) situation, that is when none of the labels a, b takes value $0, \pi$, the set C_{abf} is six- dimensional in $G \times G$, while the corresponding brane is three- dimensional in Q. One can parametrize conjugacy classes as

$$u = \mathrm{c}_a \, \mathbf{1} + \mathrm{i} \mathrm{s}_a \, \bar{n}_u \times \bar{\sigma} \; \in \; C_a \,, \tag{3.11}$$

where $\mathrm{c}_a \equiv \cos a$ and $\mathrm{s}_a \equiv \sin a$. With an analogous parametrization for $v \in C_b$, we need for $g_1 = uw$ and $g_2 = vw$ the following conditions,

$$\cos\psi_1 = \mathrm{s}_a\, \bar{n}_u \cdot \bar{n}_w \qquad\qquad \sin\psi_1\, \bar{n}_1 = \mathrm{c}_a\, \bar{n}_w + \mathrm{s}_a\, \bar{n}_u \times \bar{n}_w \qquad (3.12)$$
$$\cos\psi_2 = \mathrm{s}_b\, \bar{n}_v \cdot \bar{n}_w \qquad\qquad \sin\psi_2\, \bar{n}_2 = \mathrm{c}_b\, \bar{n}_w + \mathrm{s}_b\, \bar{n}_v \times \bar{n}_w\,.$$

We immediately observe that in order for $(g_1, g_2) \in C_{abf}$, it is necessary that $\cos\psi_1 \leq |\sin a|$, and that $\cos\psi_2 \leq |\sin b|$. Assume that $\cos\psi_1 < |\sin a|$, and that $\cos\psi_2 < |\sin b|$, which is possible because we required a, b to be different from $0, \pi$. (This implies $\bar{n}_u \cdot \bar{n}_w \neq 1$ and $\bar{n}_v \cdot \bar{n}_w \neq 1$.) The rest of the argument aims to show that we now are in the interior of a 6- dimensional region.

Given allowed choice of angles ψ_1, ψ_2, we must keep the scalar products $\bar{n}_u \cdot \bar{n}_w$ and $\bar{n}_v \cdot \bar{n}_w$ fixed accordingly. We are still free to choose the vector $\bar{n}_w$ anywhere on S^2, and we are free to rotate the vectors $\bar{n}_u \times \bar{n}_w$ and $\bar{n}_v \times \bar{n}_w$ around the axis $\bar{n}_w$.

Let us decide for a certain $\bar{n}_1$. With the freedom we have, we can certainly find vectors $\bar{n}_u$ and $\bar{n}_w$ such that $\cos a\, \bar{n}_w + \sin a\, \bar{n}_u \times \bar{n}_w$ points in the direction of $\bar{n}_1$. Since uw is a group element, it is automatic that $|\sin\psi_1 \bar{n}_1| = |\cos a\, \bar{n}_w + \sin a\, \bar{n}_u \times \bar{n}_w|$. Thus, we require that the three vectors $\bar{n}_1$, $\bar{n}_w$ and $\bar{n}_u \times \bar{n}_w$ lie in the same plane, and we fix the angle between them in this plane. We may rotate this plane, thus, we may rotate $\bar{n}_w$ around the axis $\bar{n}_1$. Furthermore, the angle between these vectors is different from zero, because we assumed $\bar{n}_u \cdot \bar{n}_w \neq 1$, which implies $\bar{n}_u \times \bar{n}_w \neq 0$.

As we rotate the above mentioned plane, $\bar{n}_w$ sweeps out a (nondegenerate) circle, around each choice of $\bar{n}_w$, the vector $\bar{n}_v \times \bar{n}_w$ sweeps out another nondegenerate circle. Thus, the vector $\bar{n}_2$ is allowed to reach a 2-dimensional subset of S^2. This completes the proof; subject to the constraints $\cos\psi_1 < |\sin a|$ and $\cos\psi_2 < |\sin b|$, $g_1 = uw$ can take any value on the group, with enough freedom left to let g_2 sweep out a 3- dimensional subset of the group (varying ψ_2 and $\bar{n}_2$). Thus, the generic set $C_{abf} \subset G \times G$ is six-dimensional.

Let a, b, c be an arbitrary nondegenerate triple of labels. These branes are 3- dimensional too. We have

$$g_1 = (\mathrm{c}_a\,\mathrm{c}_c - \mathrm{s}_a\,\mathrm{s}_c\, \bar{n}_u \cdot \bar{n}_w)\mathbf{1} - \mathrm{i}\,(\mathrm{s}_a\,\mathrm{s}_c\, \bar{n}_u \times \bar{n}_w + \mathrm{c}_a\,\mathrm{s}_c\, \bar{n}_w + \mathrm{c}_c\,\mathrm{s}_a\, \bar{n}_u) \cdot \bar{\sigma}$$
$$g_2 = (\mathrm{c}_b\,\mathrm{c}_c - \mathrm{s}_b\,\mathrm{s}_c\, \bar{n}_v \cdot \bar{n}_w)\mathbf{1} - \mathrm{i}\,(\mathrm{s}_b\,\mathrm{s}_c\, \bar{n}_v \times \bar{n}_w + \mathrm{c}_b\,\mathrm{s}_c\, \bar{n}_w + \mathrm{c}_c\,\mathrm{s}_b\, \bar{n}_v) \cdot \bar{\sigma}\,.$$

So assume ψ_1, ψ_2 are within the appropriate range, and fix the scalar products $\bar{n}_u \cdot \bar{n}_w$ and $\bar{n}_v \cdot \bar{n}_w$ accordingly. The three vectors

$$\mathrm{c}_c\,\mathrm{s}_a\, \bar{n}_u\,, \quad \mathrm{c}_a\,\mathrm{s}_c\, \bar{n}_w\,, \quad \mathrm{s}_a\,\mathrm{s}_c\, \bar{n}_u \times \bar{n}_w \qquad (3.13)$$

span a polyhedron, whose one corner is at the origin, and the opposite corner is at $\bar{n}_1$. We may rotate this polyhedron around these two points, and the rest of the proof carries over analogously, showing that also these branes project to something three-dimensional in the coset Q.

Dimensionalities of degenerate branes

In [6] and [13] it was argued that symmetry- preserving branes in WZW models are non-degenerate. Nevertheless, we shall for the remainder of this note study the corresponding degenerate branes in Q. The branes $\pi_{\mathrm{Ad}}(C_{abc})$ where the second index b is degenerate were studied in [14]. To interpret these branes as branes in the Virasoro minimal model is not without complications. In order to do that, we need to fix one of the levels to the value $l = 1$, which forbids us to use results for the large level limit. At small level, the branes are smeared around nondegenerate conjugacy classes [6, 13]. Nevertheless, let us take $b = 0$ (the case $b = \pi$ is analogous).

$$u = \mathrm{c}_a\,\mathbf{1} + \mathrm{i}\,\mathrm{s}_a\,\bar{n}_u \cdot \bar{\sigma} \qquad\qquad v = \mathbf{1} \qquad\qquad w = \mathrm{c}_c\,\mathbf{1} + \mathrm{i}\,\mathrm{s}_c\,\bar{n}_w \cdot \bar{\sigma}$$
$$g_1 = (\mathrm{c}_a\,\mathrm{c}_c - \mathrm{s}_a\,\mathrm{s}_c\,\bar{n}_u \cdot \bar{n}_w)\mathbf{1} - \mathrm{i}\,(\mathrm{s}_a\,\mathrm{s}_c\,\bar{n}_u \times \bar{n}_w + \mathrm{c}_a\,\mathrm{s}_c\,\bar{n}_w + \mathrm{c}_c\,\mathrm{s}_a\,\bar{n}_u)\cdot\bar{\sigma}$$
$$g_2 = \mathrm{c}_c\mathbf{1} - \mathrm{i}\,\mathrm{s}_c\,\bar{n}_w \cdot \bar{\sigma}$$

We immediately see that the branes are localized along the slice $\psi_2 = c$. The last of the above equations fixes $\bar{n}_w = \bar{n}_2$. As before, ψ_1 has a certain allowed range. For fixed ψ_1, note that the scalar product $\bar{n}_u \cdot \bar{n}_w$ is fixed. Thus, $\bar{n}_1$ takes a one- dimensional subset of values on S^2. To sum up, the set C_{a0c} is four-dimensional. If we project out the coordinates θ_1 and ϕ_1 on the set C_{a0c}, then $\bar{n}_2$ takes values in a circle. Since we need to project out ϕ_2, the coordinate θ_2 is fixed. Thus, these branes $\pi_{\mathrm{Ad}}(C_{a0c})$ are at most one-dimensional, extending only in the coordinate ψ_1.

If we also take $a = 0$, $g_1 = \mathrm{c}_c\mathbf{1} - \mathrm{i}\,\mathrm{s}_c\,\bar{n}_w \cdot \bar{\sigma}$, also ψ_1 is fixed; $\psi_1 = c = \psi_2$. Also note that $\bar{n}_1 = \bar{n}_w = \bar{n}_2$. Thus, $C_{00c} \subset G \times G$ is two-dimensional and must project to a region where the stabilizer is at least one-dimensional, that is at the boundary of Q, which is at $\theta = 0, \pi$. Thus, the branes $\pi_{\mathrm{Ad}}(C_{00c})$ are pointlike.

If we consider C_{a00}, we have $g_1 = \mathbf{1}\mathrm{c}_a - \mathrm{i}\,\mathrm{s}_a\,\bar{n}_u \cdot \bar{\sigma}$, $g_2 = \mathbf{1}$. This projects to a point $\psi_1 = a$, $\psi_2 = 0$ on the edge of Q.

The case left to consider is C_{ab0}. We have $g_1 = \mathrm{c}_a\,\mathbf{1} - \mathrm{i}\,\mathrm{s}_a\,\bar{n}_u \cdot \bar{\sigma}$, and $g_2 = \mathrm{c}_b\,\mathbf{1} - \mathrm{i}\,\mathrm{s}_b\,\bar{n}_v \cdot \bar{\sigma}$. Thus, C_{ab0} is four-dimensional in the covering, and projects to a line that meets the boundary of the coset at both ends $\theta = 0, \pi$.

4. Conclusions and outlook

In the non- simply connected Lie group $\mathrm{SO}(3) = \mathrm{SU}(2)/\mathbb{Z}_2$, there are two fractional branes supported at the unorientable conjugacy class $\mathbb{RP}^2 \subset \mathrm{SO}(3)$. The fundamental group of this conjugacy class is $\mathbb{Z}_2$, and the two branes correspond to different choices of gerbe modules [2]. In the CFT description, these fractional branes correspond to boundary states that are labelled by the fixed point of the simple current group $\mathbb{Z}_2$ of the SU(2) WZW model. In the model (1.1) that was investigated here, we have a comparable situation: The CFT description at levels $k, l, k+l$ involves a $\mathbb{Z}_2$ identification group, with the generator acting as

$$(a, b, c) \mapsto (k{-}a, l{-}b, k{+}l{-}c). \tag{4.1}$$

In this paper, the fixed point boundary states have been shown to be supported at the whole target. Other branes reach at most one of the singular points (corners). If there is a corresponding gerbe (module) structure for cosets, it seems to be unknown in the literature. Hopefully, the present investigations can be used as a guideline for future work in this direction.

Acknowledgments

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